

AD-A166 602 A REVIEW OF PHASE-ONLY SIDELobe NULLING INVESTIGATIONS  
AT RADC (ROME AIR DEVELOPMENT CENTER)(U) ROME AIR  
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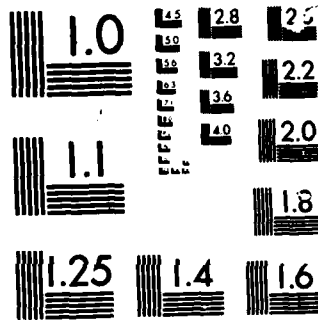
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**RADC-TR-85-145**  
**In-House Report**  
**August 1985**



# ***A REVIEW OF PHASE-ONLY SIDELobe NULLING INVESTIGATIONS AT RADC***

**Robert A. Shore**

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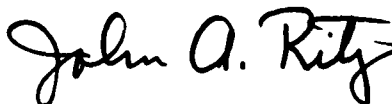
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## A Review of Phase-only Sidelobe Nulling Investigations at RADC

### 1. INTRODUCTION

The purpose of this paper is to review work performed at RADC on the subject of phase-only null synthesis and adaptive nulling in the sidelobes of linear array antenna patterns. Related investigations outside RADC are also summarized.

Interest in the subject of phase-only pattern control of array antennas has been stimulated by the growing importance of phased array antennas, since the required phase controls are already available as part of a beam steering system. The literature on the subject is fairly sizeable, as the Bibliography indicates, yet the results to date leave some basic questions unanswered. Phase-only null synthesis presents analytic and computational difficulties not present when both the amplitude and phase of the element weights can be freely perturbed. The principal source of the difficulties is that restriction of the weight perturbations to phases only makes the nulling problem nonlinear in general and not solvable analytically. This contrasts with combined phase and amplitude control where the pattern is a linear combination of the complex array weights. Indeed, it is possible that questions such as "How many nulls can be controlled in a pattern with phase-only weight variation?" cannot be answered in general. This is indicated by the fact that it is simple to construct examples of arrays in which phase-only

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nulling is impossible; consider an array where the amplitude of some one element exceeds the sum of the amplitudes of the other elements.

## 2. PHASE-ONLY SIDELOBE SYNTHESIS

Most of the phase-only control null synthesis investigations performed at RADC have been conducted within the following frame. We consider a linear array of  $N$  equispaced, isotropic elements with inter-element spacing  $d$  and phase reference at the array center. The pattern of the array is given by

$$f(u) = \sum_{n=1}^N w_n e^{j d_n u} ,$$

where  $w_n$  is the complex element excitation,

$$d_n = \frac{N-1}{2} - (n-1) , \quad n = 1, 2, \dots, N ,$$

and

$$u = \frac{2\pi}{\lambda} d \sin \theta ,$$

where  $\lambda$  is the wavelength, and  $\theta$  the pattern angle measured from broadside to the array. In pattern null synthesis we start from a given original pattern  $f_o(u)$ , with desired main beam and average sidelobe level, corresponding to the original complex element weights  $w_{on} = a_n \exp(j\phi_{on})$ . The pattern is assumed real so that the amplitudes are even-symmetric and the initial phases odd-symmetric:

$$a_{N+1-n} = a_n , \quad \phi_{o, N+1-n} = -\phi_{on} , \quad n = 1, 2, \dots, N .$$

The initial phases are generally given by

$$\phi_{on} = -d_n u_o , \quad n = 1, 2, \dots, N$$

to steer the main beam in the direction  $u = u_o$ . It is then desired to find weight perturbations of the form  $\exp(j\phi_n)$  so that the resulting pattern will have nulls at a set of  $M$  specified sidelobe directions,  $u_m$ ,  $m = 1, 2, \dots, M$ :

$$f(u_m) = \sum_{n=1}^N a_n \exp [j(\phi_{on} + \phi_n)] \exp (jd_n u_m) = 0 \quad , \quad m = 1, 2, \dots, M \quad . \quad (1)$$

In addition to the objective of imposing nulls in certain directions, a second objective is also important; namely, the desirable features of the original pattern (gain, beamwidth, main beam direction, average sidelobe level) should not be unduly degraded by the process of imposing pattern nulls. Preservation of pattern integrity demands that the perturbations of the array weights required to place nulls be kept as small as possible. This is done by requiring that the phase perturbations satisfying the null equations (1) also minimize a weighted sum of the squares of the absolute values of the element weight perturbations:

$$\begin{aligned} F &= \sum_{n=1}^N c_n |w_n - w_{on}|^2 \\ &= \sum_{n=1}^N c_n a_n^2 |e^{j\phi_n} - 1|^2 \\ &= 2 \sum_{n=1}^N c_n a_n^2 (1 - \cos \phi_n) \\ &= 4 \sum_{n=1}^N c_n a_n^2 \sin^2(\phi_n/2) \quad . \end{aligned} \quad (2)$$

The real, positive, and (even) symmetric weighting coefficients,  $\{c_n\}$ , add flexibility to the minimization of the phase perturbations and, as will be seen below, influence the shape of the resultant pattern.

Obtaining the phase perturbations that satisfy the null equations (1) subject to minimizing the objective function  $F$ , given by Eq. (2), is a nonlinear problem that does not necessarily have a solution. When a solution exists, however, the phase perturbations are odd-symmetric,<sup>1, 2</sup>

1. Shore, R. A. (1983) A Proof of the Odd-Symmetry of the Phases for Minimum Phase-Only Null Synthesis, RADC-TR-83-96, AD A130513.
2. Shore, R. A. (1984) A proof of the odd-symmetry of the phases for minimum weight perturbation phase-only null synthesis, IEEE Trans. Antennas Propaga. AP-32:528-530.

$$\phi_{N+1-n} = -\phi_n, \quad n = 1, 2, \dots, N,$$

so that the null equations, Eq. (1), can be written as

$$\sum_{n=1}^{N/2} a_n \cos(\phi_{on} + \phi_n + d_n u_m) = 0, \quad m = 1, 2, \dots, M, \quad N \text{ even}$$

$$\sum_{n=1}^{(N-1)/2} a_n \cos(\phi_{on} + \phi_n + d_n u_m) + \frac{1}{2} a_{(N+1)/2} = 0, \quad m = 1, 2, \dots, M, \quad N \text{ odd}$$

and the objective function, Eq. (2), as

$$F = 8 \sum_{n=1}^{N/2} c_n a_n^2 \sin^2(\phi_n/2), \quad N \text{ even}$$

$$F = 8 \sum_{n=1}^{(N-1)/2} c_n a_n^2 \sin^2(\phi_n/2), \quad N \text{ odd}.$$

The phase perturbations can be written in the form<sup>3</sup>

$$\phi_n = -\phi_{on} + \text{Phase} \left[ a_n - \frac{1}{c_n} \sum_{m=1}^M b_m \exp(-j d_n u_m) \right], \quad n = 1, 2, \dots, N. \quad (3)$$

In this 'beam space' representation, the coefficients  $\{b_m\}$  are real. Equation (3) does not provide an explicit solution for the phase perturbations, since the coefficients  $\{b_m\}$  are defined in terms of unknown Lagrangian multipliers. Nevertheless, this form of the phase perturbations is useful because: (a) it makes clear that the nulling problem is of dimensionality  $M$  only, (b) it makes possible an interpretation of phase-only nulling in terms of cancellation beams, (c) it can serve as the basis of a numerical method for calculating the phase perturbations, and (d) it can be incorporated into adaptive algorithms for phase-only nulling.

3. Shore, R.A. (1983) The Use of a Beam Space Representation and Nonlinear Programming in Phase-Only Nulling, RADC-TR-83-124, AD A131365.

Regarding the cancellation beam interpretation of phase-only nulling, starting with the general representation, (3), and assuming that the phase perturbations are sufficiently small so that the approximations  $\tan^{-1}(\phi_n) \approx \phi_n$  and  $\exp(j\phi_n) - 1 \approx j\phi_n$  are reasonable, the perturbed pattern can be expressed<sup>3</sup> as the sum of the original pattern and a cancellation pattern closely approximated by

$$\Delta f(u) = -\frac{1}{2} \sum_{m=1}^M b_m \sum_{n=1}^N \frac{1}{c_n^i} \left[ e^{jd_n(u-u_m)} - e^{jd_n(u-2u_o+u_m)} \right],$$

where

$$c_n^i = c_n - \frac{1}{a_n} \sum_{m=1}^M b_m \cos [d_n(u_m - u_o)] \quad , \quad n = 1, 2, \dots, N \quad .$$

The cancellation pattern for small phase perturbations is thus approximately the sum of  $M$  pairs of beams, one member of each pair directed towards an imposed null location  $u = u_m$ , and the other member, of opposite sign, directed towards the location  $u = 2u_o - u_m = u_m - 2(u_m - u_o)$ , symmetric to the null location with respect to the main beam. The shape of the cancellation beams is determined by the  $\{c_n^i\}$ . This beam space representation is still, in general, a nonlinear superposition of cancellation beams, since the beam coefficients  $\{b_m\}$  enter into the expression for the  $\{c_n^i\}$ . Only if the beam coefficients are negligibly small compared to the  $\{c_n a_n\}$  does  $c_n^i \approx c_n$  and the representation become a linear superposition of cancellation beams whose shape is determined by the choice of the  $\{c_n\}$ . For  $c_n = 1$ ,  $n = 1, 2, \dots, N$ , the beams are of the form  $\sin(Nu/2)/\sin(u/2)$ ; that is, beams corresponding to an array of  $N$  elements with uniform amplitude. For other choices of the  $\{c_n\}$ , the cancellation beams correspond to arrays with tapered amplitude distributions. For example, choosing  $c_n = 1/a_n$ ,  $n = 1, 2, \dots, N$ , results in cancellation beams of the same form as the original pattern.

The small phase perturbation representation of the cancellation pattern as a linear superposition of paired beams can be derived directly<sup>4, 5</sup> along with an explicit solution for the beam coefficients, by substituting the linear phase

4. Shore, R. A., and Steyskal, H. (1982) Nulling in Linear Array Patterns With Minimization of Weight Perturbations, RADC-TR-82-32, AD A118695.

5. Shore, R. A. (1982) A unified treatment of nulling in linear array patterns with minimized weight perturbations, IEEE-AP-S Int. Symp., 1982 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 703-706.

approximation  $\exp(j\phi) \approx 1 + j\phi$  in the null equations (1) and the objective function  $F$  of Eq. (2). From this we obtain the linearized problem of finding the set of phase perturbations that satisfy the equations

$$\sum_{n=1}^N a_n [\sin(d_n u_m + \phi_{on})] \phi_n = f_o(u_m) \quad , \quad m = 1, 2, \dots, M \quad (4)$$

and minimize

$$F = \sum_{n=1}^N c_n a_n^2 \phi_n^2 \quad . \quad (5)$$

The special case where  $c_n \approx 1/a_n^2$  is discussed by Steyskal.<sup>6,7</sup> It is also shown that the linearization method can be extended to problems involving moderate size phase perturbations by employing the technique iteratively.<sup>8</sup>

The phase perturbations that satisfy Eq. (4) and minimize Eq. (5) are

$$\phi_n = \frac{1}{a_n c_n} \sum_{m=1}^M b_m \sin(d_n u_m + \phi_{on}) \quad , \quad n = 1, 2, \dots, N \quad .$$

The vector  $\underline{b}$  of beam coefficients  $\{b_m\}$  is given by

$$\underline{b} = (A C^{-1} A^T)^{-1} \underline{f_o} \quad ,$$

where  $A$  is the  $M \times N$  matrix with elements  $A_{mn} = a_n \sin(d_n u_m + \phi_{on})$ ,  $C$  the  $N \times N$  diagonal matrix with elements  $C_{nn} = c_n a_n^2$ , and  $\underline{f_o}$  the  $M$  element column vector with elements  $f_o(u_m)$ ,  $m = 1, 2, \dots, M$ . Figure 1(a) shows the perturbed pattern of a 41-element, 40-dB Chebyshev array with a null imposed at the location  $15.23^\circ$  and the  $\{c_n\}$  chosen equal to 1. The associated cancellation pattern is shown in Figure 1(b). In contrast, Figures 2(a) and 2(b) show the corresponding patterns of the same nulling example when the  $\{c_n\}$  are chosen equal to  $1/a_n^2$ . Note the narrow

6. Steyskal, H. (1982) Simple method for pattern nulling by phase only, IEEE AP-S Int. Symp., 1982 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 707-710.
7. Steyskal, H. (1983) Simple method for pattern nulling by phase perturbation, IEEE Trans. Antennas Propaga. AP-31:163-166.
8. Shore, R. A. (1982) An Iterative Phase-Only Nulling Method, RADC-TR-82-49, AD A116949.

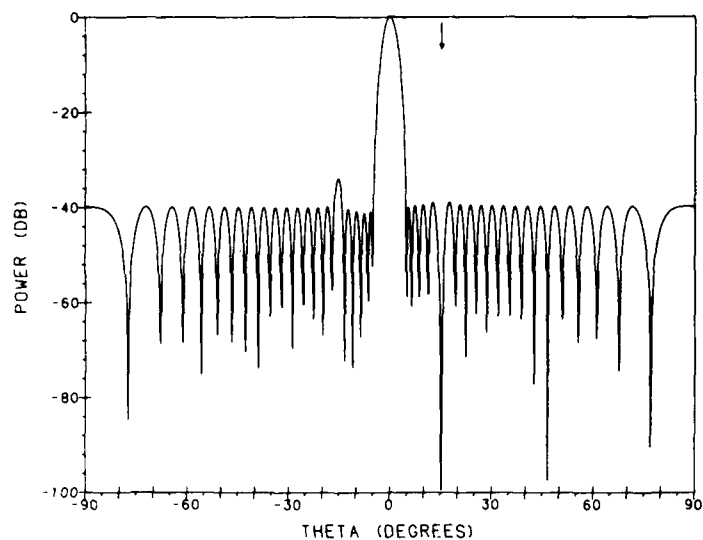


Figure 1a. Perturbed 41-Element Pattern With One Null Imposed at  $15.23^\circ$  With Phase-Only Perturbations Minimizing  $\Sigma(a_n \phi_n)^2$ .  $\theta = -90^\circ$  to  $+90^\circ$

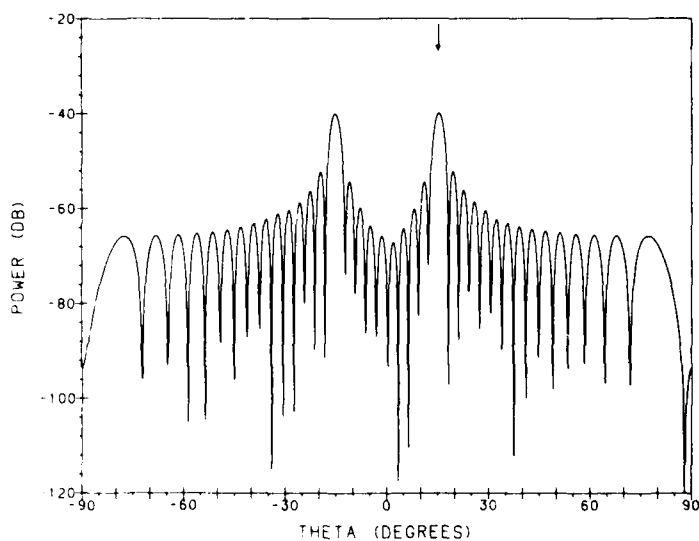


Figure 1b. 41-Element Cancellation Pattern to Impose One Null at  $15.23^\circ$  With Phase-Only Perturbations, Minimizing  $\Sigma(a_n \phi_n)^2$ .  $\theta = -90^\circ$  to  $+90^\circ$

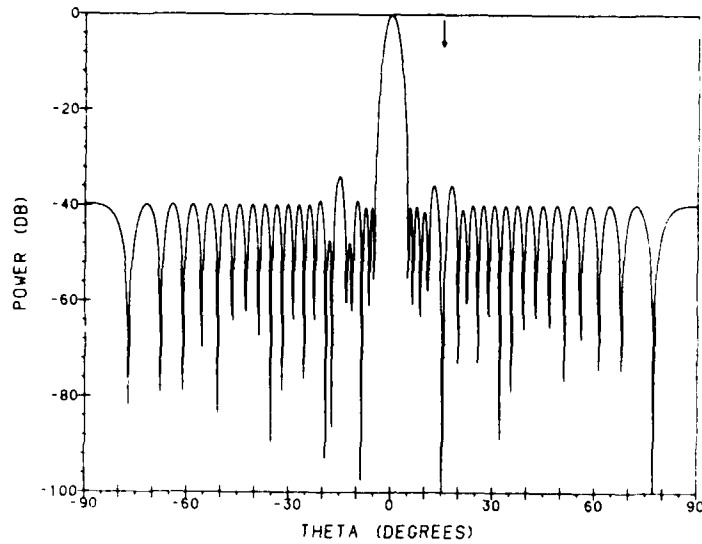


Figure 2a. Perturbed 41-Element Pattern With One Null Imposed at 15.23° With Phase-Only Perturbations Minimizing  $\sum \phi_n^2$ .  $\theta = -90^\circ$  to  $+90^\circ$

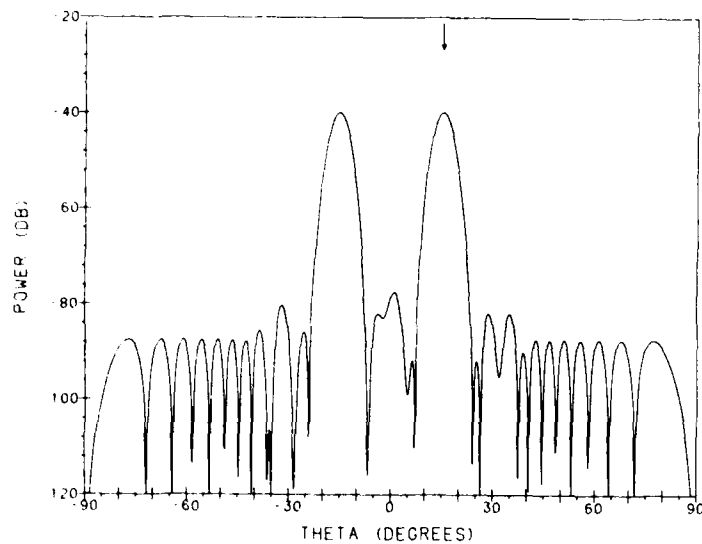


Figure 2b. 41-Element Cancellation Pattern to Impose One Null at 15.23° With Phase-Only Perturbations, Minimizing  $\sum \phi_n^2$ .  $\theta = -90^\circ$  to  $+90^\circ$

main beam and relative high sidelobes of the cancellation pattern of Figure 1(b), and the broad main beam but very low sidelobes of the cancellation pattern of Figure 2(b) corresponding to the pattern of an array whose amplitudes are the squares of a 40-dB Chebyshev amplitude distribution. In both examples the perturbed pattern shows an approximately 6-dB increase at the location  $-15.23^\circ$  symmetric to that of the imposed null, resulting from the auxiliary beam of the cancellation pattern adding in phase to the original pattern.

Returning to the general (that is, not necessarily small phase perturbation) phase-only nulling problem, we have seen above that the phase perturbations to impose nulls at the  $M$  locations  $u = u_m$ ,  $m = 1, 2, \dots, M$ , subject to minimizing the objective function  $F$  of Eq. (2), cannot be calculated directly from the representation (3). However, this representation can be used as the basis for efficiently calculating the phase perturbations, since the beam coefficients can be obtained by using computer algorithms developed for solving the so-called nonlinear programming problem—the problem of minimizing or maximizing a nonlinear function of several variables subject to a set of nonlinear equality and/or inequality constraints.<sup>3</sup> Such nonlinear optimization computer methods must be used to calculate the phase perturbations when they are large, as for example, when a null is imposed in the main beam vicinity, when multiple nulls are imposed within a relatively narrow angular sector, or when the number  $M$  of imposed nulls increases beyond  $M \ll N$ . Computational difficulties can arise if too many nulls are desired within a small angular pattern sector. The beam coefficients grow large in magnitude and alternate in sign as the number of nulls increases, and increasingly large changes in the beam coefficients are required to bring about small changes in the objective function and the constraint functions. Viewed intuitively in terms of a picture of cancellation beams, this behavior of the beam coefficients is caused by an interference of the main lobes of the cancellation beams with one another, thus requiring extensive mutual adjustment of the magnitudes of the beams for nulling to occur. For widely spaced nulls, the cancellation beams interact only through their sidelobes, and beam coupling is negligible.

Nonlinear programming techniques<sup>9,10</sup> can also be used to obtain the phase perturbations directly, rather than obtaining them via the beam space representation (3). The unknowns are then the phase perturbations themselves instead of the beam coefficients, and the number of unknowns is, of course,  $N/2$  or  $(N-1)/2$  for an even or odd number of array elements respectively.

9. Shore, R.A. (1983) Phase-Only Nulling as a Nonlinear Programming Problem, RADC-TR-83-37, AD A130552.

10. Shore, R.A. (1983) The use of nonlinear programming techniques for phase-only null synthesis, IEEE AP-S Int. Symp., 1983 Int. Symp. Digest, Antennas and Propagation, Vol. I, pp. 207-210.



An interesting application of phase-only nulling, with no restrictions on the size of the phase perturbations, is the imposing of nulls at pairs of locations symmetric with respect to the main beam. As seen above, null placement with small phase perturbations results in an auxiliary beam directed at the location symmetric to the null location which adds in phase to the original pattern; thus, it is impossible to null at symmetric pattern locations with small phase perturbations. This conclusion can also be established with a simple analytic argument.<sup>11, 12</sup> (It is important to note, however, that this conclusion is valid only for the ideal patterns we are considering here. For realistic arrays involving phase and amplitude errors and complex pattern values, it is indeed possible to null at symmetric pattern locations with small phase perturbations.<sup>13</sup>) If the restriction that the phase perturbations be small is removed, then it is possible to null at symmetric pattern locations in ideal array patterns with phase-only weight control. The phases can be calculated using nonlinear programming. Figure 3 shows an example of the patterns obtained. As can be seen in this figure, the resulting patterns are characterized by considerable distortion—a consequence of the fact that some of the phase perturbations are large. Details on the calculations and more pattern examples are given in References 11 and 12.

The phase-only nulling investigations described thus far have focussed on the problem of placing nulls at prescribed locations while minimizing the weight perturbations. In applications such as minimizing the effects of clutter or of wide bandwidth point interferences, it may be desirable to reduce sidelobes in an entire sector of the pattern. While this can be done by imposing a series of nulls in the sector, an alternative method is also of interest. As we have noted, the preservation of desirable pattern features such as gain and beamwidth, or an already low average sidelobe level, demands that the perturbations of the array weights required to achieve the lowered sidelobes be kept as small as possible. A trade-off exists between the two objectives of lowered sidelobes and the preservation of the integrity of a design antenna pattern. This suggests that a useful performance measure in sidelobe sector nulling is the weighted sum  $P$  of the squared weight perturbations and the average power in a specified sidelobe region

11. Shore, R.A. (1983) Phase-Only Nulling at Symmetric Pattern Locations, RADC-TR-83-100, AD A133128.
12. Shore, R.A. (1984) Nulling at symmetric pattern location with phase-only weight control, IEEE Trans. Antennas Propaga. AP-32:530-533.
13. Baird, C.A., and Rassweiler, G.G. (1976) Adaptive sidelobe nulling using digitally controlled phase-shifters, IEEE Trans. Antennas Propaga. AP-24:638-649.

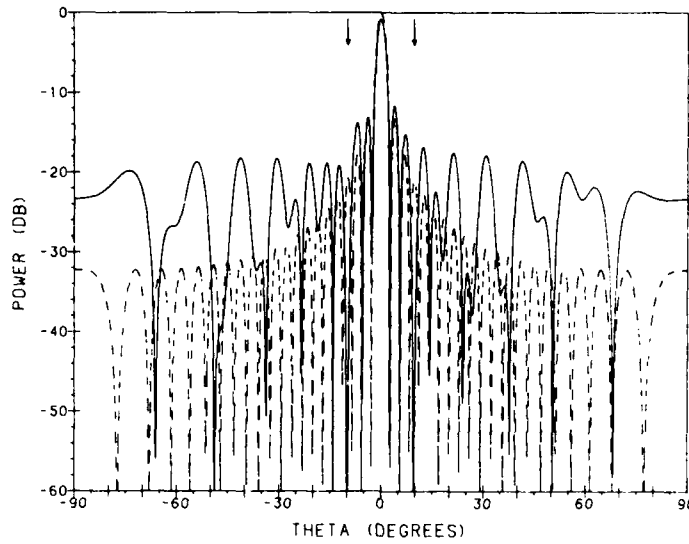


Figure 3. Original Uniform Amplitude 41-Element Pattern (-----) and Perturbed Pattern (——) With Nulls Imposed at  $\pm 9.74^\circ$ .  $\theta = -90^\circ$  to  $+90^\circ$

$$P = \mu_1 \sum_{n=1}^N |w_n - w_{on}|^2 + \mu_2 P_{av}(u_c, \epsilon) .$$

Here,  $u_c$  is the center of the sidelobe sector,  $\epsilon$  is the half-width of the sector, and  $P_{av}(u_c, \epsilon)$ , the average sidelobe sector power, is given by

$$P_{av}(u_c, \epsilon) = \frac{1}{2\epsilon} \int_{u_c - \epsilon}^{u_c + \epsilon} |f(u)|^2 du$$

$$= \sum_{n=1}^N \sum_{m=1}^N a_n a_m \cos [\phi_{on} - \phi_{om} + \phi_n - \phi_m + (d_n - d_m)u_0]$$

$$\cdot \text{sinc} [(d_n - d_m)\epsilon] ,$$

where  $\text{sinc}(x) = \sin(x)/x$ . By varying the weights  $\mu_1$  and  $\mu_2$  assigned to the weight perturbations and the average power in the sidelobe sector respectively, and minimizing the performance measure, it is then possible to shift the relative emphasis placed on the two principal objectives.

When both the amplitude and the phase of the array weights can be freely varied, an analytic solution can be obtained for the array weights that minimize the performance measure. When perturbations of the complex weights are restricted to be of the phases only, the phases that minimize the performance measure cannot be obtained analytically, but they can be found numerically by using nonlinear optimization computer codes.<sup>14, 15</sup> Figure 4 shows the pattern of an unperturbed, uniform amplitude, 41-element array and the perturbed pattern with sidelobes lowered in the angular sector  $[20^\circ, 30^\circ]$  with  $\mu_2/\mu_1 = 100$ . Figure 5 shows the corresponding patterns when  $\mu_2/\mu_1 = 100,000$ . It is interesting to note that as  $\mu_2/\mu_1 \rightarrow \infty$ , and, hence, as increasing weight is placed on lowering the sidelobes in the sector as compared to preserving the original pattern, only a relatively small number of nulls are moved into the nulling sector. For uniform amplitude arrays of 11, 21, and 41 elements respectively, a maximum of only 4, 6, and 9 nulls are placed in the sector  $[20^\circ, 30^\circ]$  as  $\mu_2/\mu_1 \rightarrow \infty$ . This behavior contrasts strongly with that found when both the amplitude and the phase of the array weights can be varied, in which case more and more nulls, up to the maximum of  $N - 1$ , are moved into the nulling sector as  $\mu_2/\mu_1 \rightarrow \infty$ .

We conclude this section by mentioning a study<sup>16</sup> made of a phase-only nulling method that is appealing because of its simplicity, but which, unfortunately, was found to be rather ineffective. The method consists of: (1) analytically obtaining the minimum complex weight perturbations that serve to impose nulls at a set of specified pattern locations, and (2) choosing the phase-only perturbations that are closest in a mean-square sense to the complex weight perturbations. The desired phase perturbations are simply the respective phases of the complex weight perturbations. A modest degree of pattern reduction at the specified null locations can be achieved by this method, but, in general, the procedure is inferior to the small phase perturbation linearization method.

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14. Shore, R.A., and Pierre, D.A. (1985) Sidelobe Sector Nulling With Minimized Phase Perturbations, RADC-TR-85-56, AD A157058.
  15. Shore, R.A., and Pierre, D.A. (1985) Sidelobe sector nulling with minimized phase perturbations, IEEE AP-S Int. Symp., 1985 Int. Symp. Digest, Antennas and Propagation, Vol. I, pp. 139-142.
  16. Shore, R.A. (1982) Phase-Only Nulling as a Least Squares Approximation to Complex Weight Nulling, RADC-TR-82-129, AD A118722.

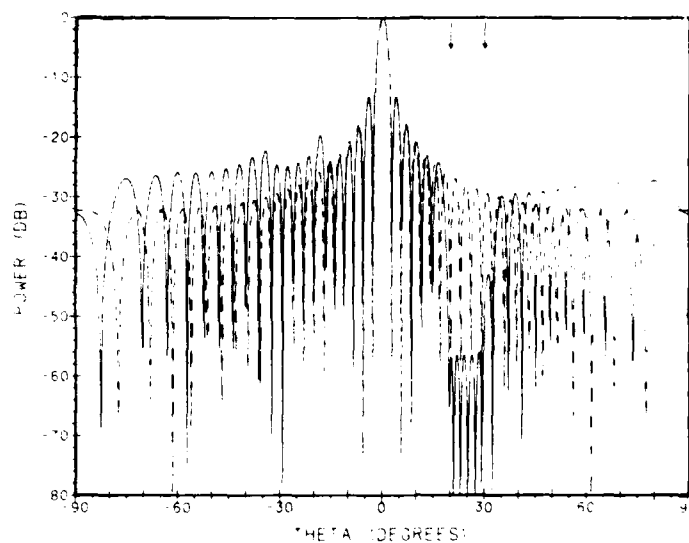


Figure 4. Unperturbed Uniform 41-Element Array Pattern (----) and Perturbed Pattern (—) With Lowered Sidelobes in the Sector  $[20^\circ, 30^\circ]$ .  $\mu_2/\mu_1 = 100$

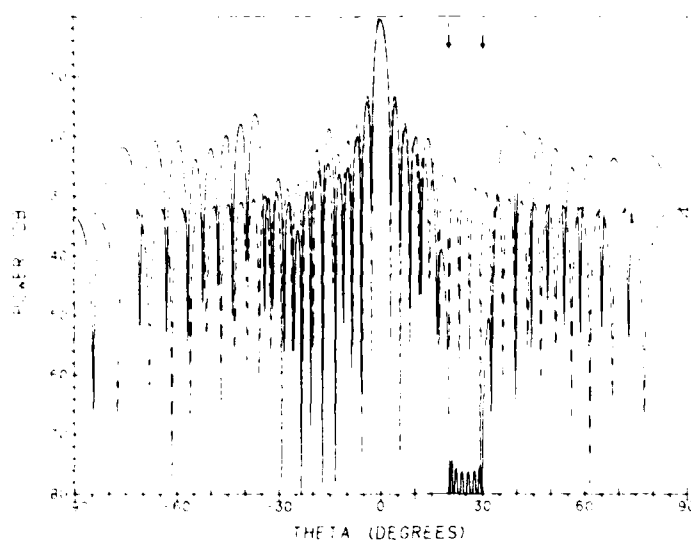


Figure 5. Unperturbed Uniform 41-Element Array Pattern (----) and Perturbed Pattern (—) With Lowered Sidelobes in the Sector  $[20^\circ, 30^\circ]$ .  $\mu_2/\mu_1 = 100,000$

### 3. ADAPTIVE PHASE-ONLY SIDELobe NULLING

In addition to the null synthesis studies summarized above, experimental investigations in adaptive phase-only nulling were also conducted. The experiments were performed on a precision linear array of 80 S-band H-plane sectoral horn radiators with a Taylor amplitude illumination taper for the sum pattern. Phase control of each element is accomplished with an 8-bit digitally controlled garnet phase shifter, making the least significant bit (LSB) about  $1.4^\circ$ . Although not designed to be adaptive, the antenna was made adaptive by linking control of the phase shifters to the HP-21MX computer in the SA-2020 computer-controlled antenna measurement system, and programming the computer to perform adaptive suppression.

Adaptive cancellation was performed with two different algorithms designed to minimize the total output power of the array. An iterative gradient search algorithm was defined by

$$\phi_{\text{NEW}}(i) = \phi_{\text{OLD}}(i) - \mu \frac{\Delta P(i)}{\Delta \phi}.$$

Here,  $\phi_{\text{NEW}}(i)$  and  $\phi_{\text{OLD}}(i)$  are the new and old phase shifter settings for the  $i^{\text{th}}$  element at each iteration,  $\Delta P(i)$  is the change in total output power resulting from a change of  $\Delta \phi$  in the phase of the  $i^{\text{th}}$  element, keeping all other phases fixed at their old values, and  $\mu$ , the gradient step size is

$$\mu = \frac{(\Delta \phi)^2}{\sqrt{\sum [\Delta P(i)]^2}}.$$

The algorithm is iterated starting with a given  $\Delta \phi$  until no further reduction in total output power is obtained; then  $\Delta \phi$  is reduced by one bit, and the procedure repeated until  $\Delta \phi$  equals 1 LSB. This algorithm may employ all elements of the array (fully adaptive) or any subset of elements (partially adaptive).

The beam space algorithm causes all phase shifters to be reset simultaneously for each power measurement rather than one at a time. Setting  $\phi(i)$ , the phase of the  $i^{\text{th}}$  element at the  $n^{\text{th}}$  measurement according to

$$\phi(i) = \sum_{m=1}^M b_{n,m} \sin [2\pi d_i (\sin \theta_s - \sin \theta_m) + \beta_{n,m}]$$

forms  $M$  cancellation beams with complex amplitudes  $b_{n,m} \exp(j\beta_{n,m})$  in the  $M$  specified interference directions,  $\theta_m$ ,  $m = 1, 2, \dots, M$ . Here  $\theta_s$  is the beam steering angle of the array, referenced to broadside along with the  $\{\theta_m\}$ , and

$d_i$  is the distance in wavelengths of the  $i^{\text{th}}$  element from the array center. The algorithm to determine the beam coefficients first brackets each phase,  $\beta_{n,m}$ , within a  $45^\circ$  sector in four measurements, with the  $\{b_{n,m}\}$  initially set to give beam amplitudes equal to the rms sidelobe level. Further iterations bracket the  $\{\beta_{n,m}\}$  into successively narrower angular regions until the convergence criterion has been met. A similar bracketing process is then used to determine the amplitudes of the beam coefficients. Note that the beam space algorithm, unlike the phase gradient method, requires a knowledge of the number and directions of the interferences.

The adaptive performance of the antenna and gradient algorithm was tested with a fully adaptive array and a variety of partially adaptive configurations using a single CW interference signal. The partially adaptive configurations, with from four to ten adaptive elements, resulted in considerably shorter adaptation times than the fully adaptive array. All but one of the configurations were effective in placing a pattern null of better than -20 dB below the quiescent pattern value. The one exception was a configuration with all adaptive elements placed on one side of the array center. Other than this case, there was little difference in the depth of null achieved by the various configurations of adaptive elements, the pattern being lowered down to essentially the noise level at the location of the interference source. Sidelobe distortion was greatest for the four-element configurations, but did not exceed 3.5 dB. There was virtually no main beam distortion. Figure 6 shows the quiescent and adapted pattern for the fully adaptive array, and Figures 7 and 8 show representative partially adaptive patterns. Adaptive nulling with the beam space algorithm was also successfully tested with the experimental array. Figure 9 shows the quiescent and adapted patterns for a CW interference located at  $47^\circ$  from broadside. Note the small pattern perturbations everywhere except near the locations  $\pm 47^\circ$ . Further details of the experiments and related computer simulations are given in References 17 and 18.

17. Haupt, R. L., and Shore, R. A. (1984) Experimentally partially adaptive nulling in a low sidelobe phased array, IEEE AP-S Int. Symp., 1984 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 823-826.
18. Haupt, R. L., O'Brien, M. J., and Shore, R. A. (1984) Using the phase shifters in an experimental array for adaptive nulling, Proc. 1984 Int. Symp. Noise and Clutter Rejection in Radars and Imaging Sensors, Tokyo, Japan, October 1984, pp. 579-584.

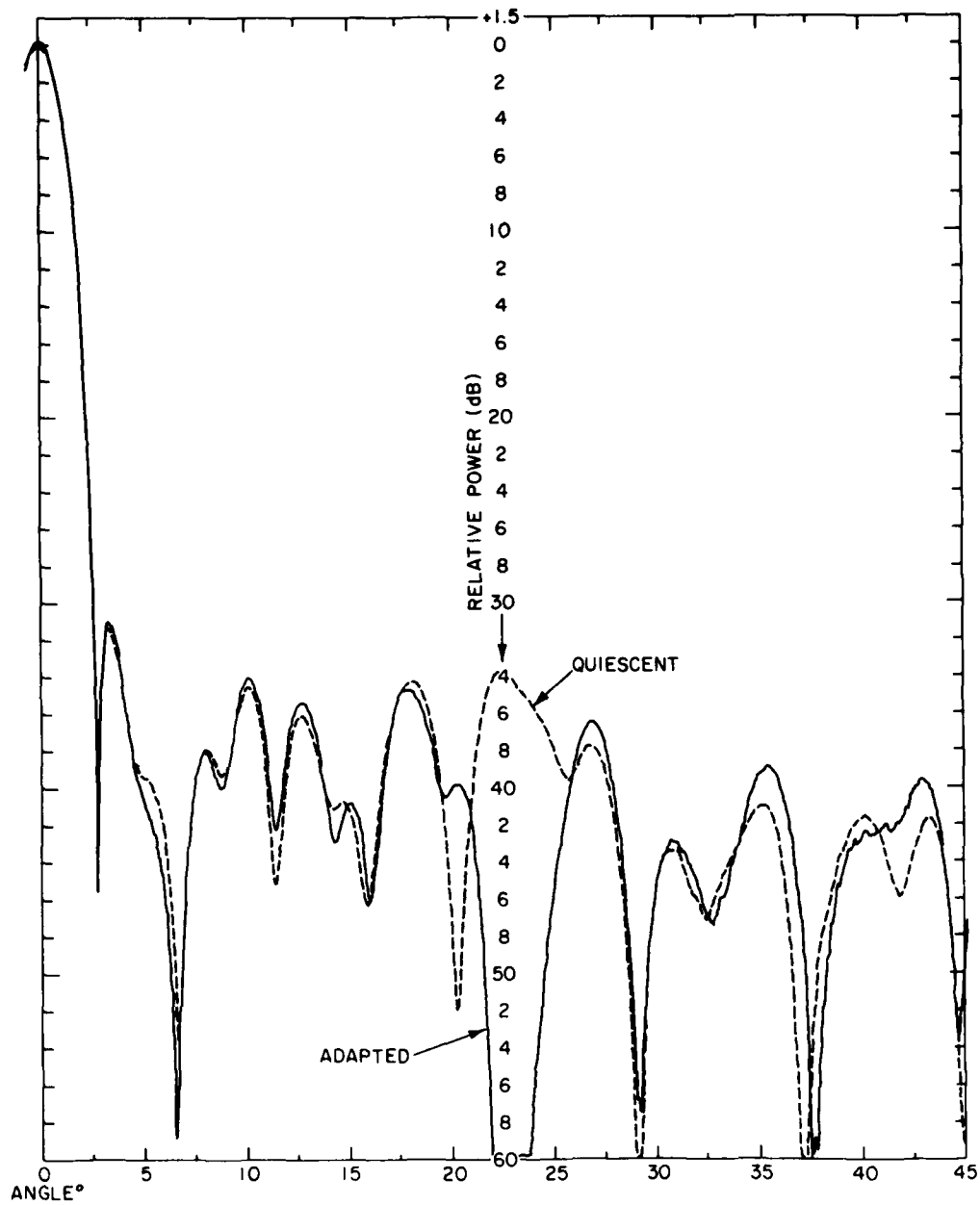


Figure 6. Measured Quiescent Pattern and Adapted Pattern Obtained With the Fully Adaptive 80-Element Array. Noise source at 22°

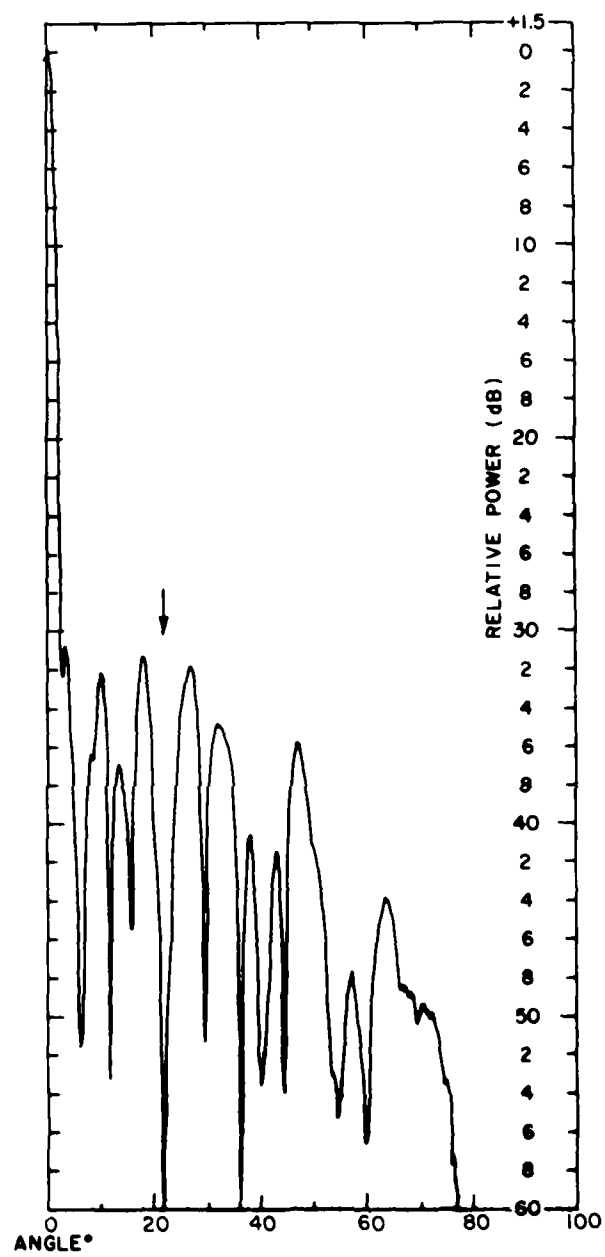


Figure 7. Measured Adapted Pattern Obtained With 10 Adaptive Elements (25-30, 51-55) and Noise Source at 22°



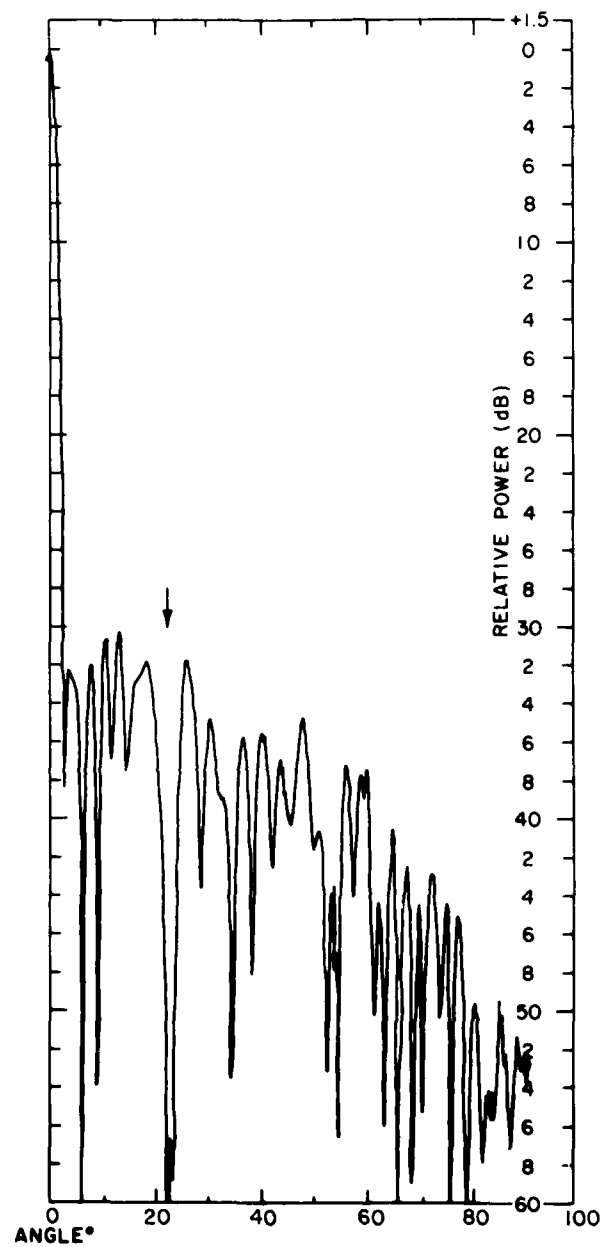


Figure 8. Measured Adapted Pattern Obtained With 4 Adaptive Elements (20, 23, 58, 61) and Noise Source at 22°

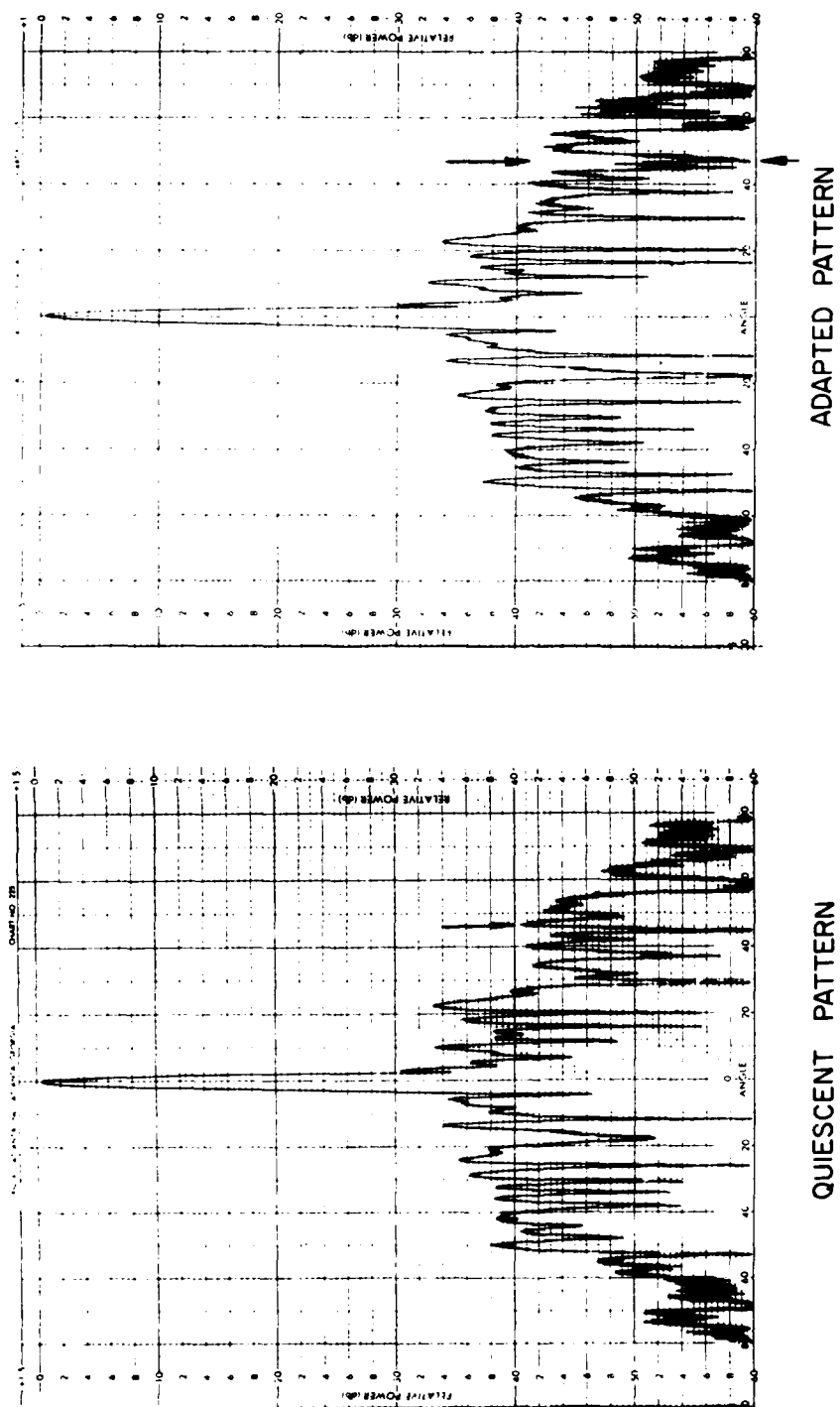


Figure 9. Measured Quiescent Pattern and Adapted Pattern Obtained With Cancellation Beam. Noise source at  $47^\circ$

#### 4. RELATED STUDIES

In this concluding section of the report we briefly summarize some other phase-only investigations closely related to those performed here.

The basic paper by Baird and Rassweiler<sup>13</sup> was the starting point for the RADC work. Baird and Rassweiler treat the problem of nulling in a uniform amplitude array subject to minimizing the difference between the array output and a desired signal. They derive a beam space representation of the phase perturbations by employing an ingenious mathematical argument that serves as the basis for the beam space representation obtained in Reference 3. Their general result is specialized to the case of small phase perturbations leading to the interpretation of the perturbed pattern as the sum of the original pattern and cancellation beams. Experimental adaptive nulling results for a 16-element array are also described.

Ananasso<sup>19,20</sup> uses the small phase perturbation solution of Baird and Rassweiler in a computer simulation to construct the paired beam form of the cancellation pattern needed to place a single null in a uniform array pattern. He then discusses the degradation of null depth produced by phase quantization.

Giusto and de Vicenti<sup>21</sup> consider the problem of synthesizing, with phase-only weight control, a series of nulls in a small angular sector of an array pattern. They start with the Baird and Rassweiler general beam space representation of the phase perturbations, extended to arrays of non-uniform amplitudes, and obtain the beam coefficients that will minimize the total array output power by using a random search or simplex method. Their method is thus very similar to that presented in Reference 3, except that no attempt is made to minimize weight perturbations.

Additional applications of phase-only weight control to pattern nulling and to other aspects of pattern control are referenced in the Bibliography.

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19. Ananasso, F. (1981) Nulling performance of null-steering array with digital phase-only weights, Electron. Lett. 17:255-257.
  20. Ananasso, F. (1981) Null steering using digital weighting, Microwave System News 11:78-94.
  21. Giusto, R., and de Vincenti, P. (1983) Phase-only optimization for the generation of wide deterministic nulls in the radiation pattern of phased arrays, IEEE Trans. Antennas Propaga. AP-31:814-817.

## References

1. Shore, R. A. (1983) A Proof of the Odd-Symmetry of the Phases for Minimum Phase-Only Null Synthesis, RADC-TR-83-96, AD A130513.
2. Shore, R. A. (1984) A proof of the odd-symmetry of the phases for minimum weight perturbation phase-only null synthesis, IEEE Trans. Antennas Propaga. AP-32:528-530.
3. Shore, R. A. (1983) The Use of a Beam Space Representation and Nonlinear Programming in Phase-Only Nulling, RADC-TR-83-124, AD A131365.
4. Shore, R. A., and Steyskal, H. (1982) Nulling in Linear Array Patterns With Minimization of Weight Perturbations, RADC-TR-82-32, AD A118695.
5. Shore, R. A. (1982) A unified treatment of nulling in linear array patterns with minimized weight perturbations, IEEE AP-S Int. Symp., 1982 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 703-706.
6. Steyskal, H. (1982) Simple method for pattern nulling by phase only, IEEE AP-S Int. Symp., 1982 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 707-710.
7. Steyskal, H. (1983) Simple method for pattern nulling by phase perturbation, IEEE Trans. Antennas Propaga. AP-31:163-166.
8. Shore, R. A. (1982) An Iterative Phase-Only Nulling Method, RADC-TR-82-49, AD A116949.
9. Shore, R. A. (1983) Phase-Only Nulling as a Nonlinear Programming Problem, RADC-TR-83-37, AD A130552.
10. Shore, R. A. (1983) The use of nonlinear programming techniques for phase-only null synthesis, IEEE AP-S Int. Symp., 1983 Int. Symp. Digest, Antennas and Propagation, Vol. I, pp. 207-210.
11. Shore, R. A. (1983) Phase-Only Nulling at Symmetric Pattern Locations, RADC-TR-83-100, AD A133128.
12. Shore, R. A. (1984) Nulling at symmetric pattern location with phase-only weight control, IEEE Trans. Antennas Propaga. AP-32:530-533.

13. Baird, C.A., and Rassweiler, G.G. (1976) Adaptive sidelobe nulling using digitally controlled phase-shifters, IEEE Trans. Antennas Propaga. AP-24:638-649.
14. Shore, R.A., and Pierre, D.A. (1985) Sidelobe Sector Nulling With Minimized Phase Perturbations, RADC-TR-85-56, AD A157058.
15. Shore, R.A., and Pierre, D.A. (1985) Sidelobe sector nulling with minimized phase perturbations, IEEE AP-S Int. Symp., 1985 Int. Symp. Digest, Antennas and Propagation, Vol. I, pp. 139-142.
16. Shore, R.A. (1982) Phase-Only Nulling as a Least Squares Approximation to Complex Weight Nulling, RADC-TR-82-129, AD A118722.
17. Haupt, R.L., and Shore, R.A. (1984) Experimentally partially adaptive nulling in a low sidelobe phased array, IEEE AP-S Int. Symp., 1984 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 823-826.
18. Haupt, R.L., O'Brien, M.J., and Shore, R.A. (1984) Using the phase shifters in an experimental array for adaptive nulling, Proc. 1984 Int. Symp. Noise and Clutter Rejection in Radars and Imaging Sensors, Tokyo, Japan, October 1984, pp. 579-584.
19. Ananasso, F. (1981) Nulling performance of null-steering array with digital phase-only weights, Electron. Lett. 17:255-257.
20. Ananasso, F. (1981) Null steering using digital weighting, Microwave System News 11:78-94.
21. Giusto, R., and de Vincenti, P. (1983) Phase-only optimization for the generation of wide deterministic nulls in the radiation pattern of phased arrays, IEEE Trans. Antennas Propaga. AP-31:814-817.

## Bibliography

- Ananasso, F. (1981) Nulling performance of null-steering array with digital phase-only weights, Electron. Lett. 17:255-257.
- Ananasso, F. (1981) Null steering using digital weighting, Microwave System News 11:78-94.
- Baird, C.A., and Rassweiler, G.G. (1976) Adaptive sidelobe nulling using digitally controlled phase-shifters, IEEE Trans. Antennas Propaga. AP-24:638-649.
- Chakraborty, A., Das, B.N., and Sanyal, G.S. (1982) Beam shaping using non-linear phase distribution in a uniformly spaced array, IEEE Trans. Antennas Propaga. AP-30:1031-1034.
- Cheng, D.K., and Raymond, P.D. (1971) Optimisation of array directivity by phase adjustments, Electron. Lett. 7:552-554.
- Cheng, D.K., and Goto, N. (1975) Proc. 1975 European Microwave Conference, p. 443. Reprinted in Advanced Antenna Technology, P.J.B. Clarricoats (Ed.), Microwave Exhibitions and Publishers, Ltd., 1981, pp. 34-38.
- Giusto, R., and de Vincenti, P. (1983) Phase-only optimization for the generation of wide deterministic nulls in the radiation pattern of phased arrays, IEEE Trans. Antennas Propaga. AP-31:814-817.
- Guo, Y.C., and Smith, M.S. (1981) Phase weighting for linear antenna beams, Electron. Lett. 17:121-122.
- Haupt, R.L. (1984) Adaptive nulling in monopulse antennas, IEEE AP-S Int. Symp., 1984 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 819-822.
- Haupt, R.L., and Shore, R.A. (1984) Experimentally partially adaptive nulling in a low sidelobe phased array, IEEE AP-S Int. Symp., 1984 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 823-826.
- Haupt, R.L., O'Brien, M.J., and Shore, R.A. (1984) Using the phase shifters in an experimental array for adaptive nulling, Proc. 1984 Int. Symp. Noise and Clutter Rejection in Radars and Imaging Sensors, Tokyo, Japan, October 1984, pp. 579-584.

- Hirasawa, K. (1985) Phase-only optimization of phased array excitation by bi-quadratic programming, Phased Arrays 1985 Symposium--Proceedings, H. P. Steyskal (Ed.), RADC-TR-85-171, pp. 147-156.
- Hockham, G. A., et al (1980) Null-steering techniques for application to large array antennas, Conf. Proc., Military Microwaves 80, October 1980, pp. 623-628. Reprinted in Advanced Antenna Technology, P.J.B. Clarricoats (Ed.), Microwave Exhibitions and Publishers, Ltd., 1981, pp. 156-161.
- Kikuma, N., and Takao, K. (1985) A directionally constrained adaptive array with phase-only control, Trans. IECE Japan E 68:325-330.
- Leavitt, M.K. (1976) A phase adaptation algorithm, IEEE Trans. Antennas Propaga. AP-24:754-756.
- Lee, J.J., et al (1980) An adaptive pillbox array antenna, IEEE AP-S Int. Symp., 1980 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 375-378.
- Mendelovicz, E., and Oestreich, E. T. (1979) Phase-only adaptive nulling with discrete values, IEEE AP-S Int. Symp., 1979 Int. Symp. Digest, Antennas and Propagation, Vol. I, pp. 193-198.
- Mendelovicz, E., and Oestreich, E. T. (1979) Phase-only adaptive nulling with discrete values, Proc. 1979 European Microwave Conference, p. 164. Reprinted in Advanced Antenna Technology, P.J.B. Clarricoats (Ed.), Microwave Exhibitions and Publishers, Ltd., 1981, pp. 79-83.
- Perry, J. L. (1984) Modern radar clutter suppression techniques: a comparison of theoretical and measured results, Proc. Int. Radar Conf., Paris, pp. 485-491.
- Roger, J., et al (1977) An adaptive phased array antenna using simple algorithm, Proc. 1977 European Microwave Conference, p. 297. Reprinted in Advanced Antenna Technology, P.J.B. Clarricoats (Ed.), Microwave Exhibitions and Publishers, Ltd., 1981, pp. 59-63.
- Shchapov, Y. M. (1983) A random-search algorithm in the problem of the adaptation of an antenna array with discrete phase shifters, Izvestiya VUZ. Radioelektronika 26:80-85.
- Shore, R. A. (1982) An Iterative Phase-Only Nulling Method, RADC-TR-82-49, AD A116949.
- Shore, R. A. (1984) Nulling at symmetric pattern location with phase-only weight control, IEEE Trans. Antennas Propaga. AP-32:530-533.
- Shore, R. A. (1983) On the Odd-Symmetry of Minimum Phase-Only Perturbations, RADC-TR-83-26, AD A130514.
- Shore, R. A. (1982) Phase-Only Nulling as a Least Squares Approximation to Complex Weight Nulling, RADC-TR-82-129, AD A118722.
- Shore, R. A. (1983) Phase-Only Nulling as a Nonlinear Programming Problem, RADC-TR-83-37, AD A130552.
- Shore, R. A. (1983) Phase-Only Nulling at Symmetric Pattern Locations, RADC-TR-83-100, AD A133128.
- Shore, R. A. (1984) A proof of the odd-symmetry of the phases for minimum weight perturbation phase-only null synthesis, IEEE Trans. Antennas Propaga. AP-32:528-530.
- Shore, R. A. (1983) A Proof of the Odd-Symmetry of the Phases for Minimum Weight Perturbation, Phase-Only Null Synthesis, RADC-TR-83-96, AD A130513.
- Shore, R. A. (1982) A unified treatment of nulling in linear array patterns with minimized weight perturbations, IEEE AP-S Int. Symp., 1982 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 703-706.

- Shore, R.A. (1983) The Use of a Beam Space Representation and Nonlinear Programming in Phase-Only Nulling, RADC-TR-83-124, AD A131365.
- Shore, R.A. (1983) The use of nonlinear programming techniques for phase-only null synthesis, IEEE AP-S Int. Symp., 1983 Int. Symp. Digest, Antennas and Propagation, Vol. I, pp. 207-210.
- Shore, R.A., and Pierre, D.A. (1985) Sidelobe sector nulling with minimized phase perturbations, IEEE AP-S Int. Symp., 1985 Int. Symp. Digest, Antennas and Propagation, Vol. I, pp. 139-142.
- Shore, R.A., and Pierre, D.A. (1985) Sidelobe Sector Nulling With Minimized Phase Perturbations, RADC-TR-85-56, AD A157058.
- Shore, R.A., and Steyskal, H. (1982) Nulling in Linear Array Patterns With Minimization of Weight Perturbations, RADC-TR-82-32, AD A118695.
- Steyskal, H. (1982) Simple method for pattern nulling by phase only, IEEE AP-S Int. Symp., 1982 Int. Symp. Digest, Antennas and Propagation, Vol. II, pp. 707-710.
- Steyskal, H. (1983) Simple method for pattern nulling by phase perturbation, IEEE Trans. Antennas Propaga. AP-31:163-166.
- Steyskal, H., Haupt, R.L., and Shore, R.A. (1985) Null placement in array antenna patterns, Proc. 1985 Int. Symp. Antennas and Propagation, Japan, Vol. III, pp. 759-762.
- Steyskal, H., Haupt, R.L., and Shore, R.A. (1985) Null placement in array antenna patterns, Proc. 1985 Int. Symp. Antennas and EM Theory, China, pp. 444-449.
- Thompson, P.A. (1976) Adaptation by direct phase-shift adjustment in narrow-band adaptive antenna systems, IEEE Trans. Antennas Propaga. AP-24:756-760.
- Turner, R.M. (1977) Null placement and antenna pattern synthesis by control of the element steering phases of a phased-array radar, Conf. Proc., Int. Conf. RADAR-77, London, October 1977, pp. 222-225.
- Turner, R.M., et al (1978) A steering control architecture and its use for null steering in a phased-array radar, Conf. Proc., Int. Conf. on Radar, Paris, December 1978.
- Voges, R.C., and Butler, J.K. (1972) Phase optimization of antenna array gain with constrained amplitude excitation, IEEE Trans. Antennas Propaga. AP-20:432-436.



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